B. Sc. B.Ed. SEMESTER I EXAMINATION 2020

Subject: Physics

CC 1 (Mathematical Physics - I)

FULL MARKS: 50 TIME ALLOWED: 2 HOURS

Answer any **Ten** (10) questions

1. (i) Write down the expression for divergence of a vector in spherical polar and cylindrical coordinates. (2 marks)

(ii) Prove that the adjoint of a diagonal matrix is a diagonal matrix. (3 marks)

2. (i) Calculate $\frac{dy}{dx}$ if $e^{xy} + y\ln x = \cos 2x$. (2 marks)

(ii) Prove that $\frac{d^n}{dx^n}(x^2 \sin x) = [x^2 - n(n-1)]\sin(x + \frac{n\pi}{2}) - 2nx\cos(x + \frac{n\pi}{2})$ (3 marks)

3. State and prove Stokes theorem. (5 marks)

4. (i) If $f(x,y) = x^2\tan^{-1}\left(\frac{y}{x}\right)$, then evaluate $\frac{\partial^2 f}{\partial x \partial y}$ at the point (1,1). (2 marks)

(ii) Prove that $\boldsymbol{\nabla}^2 r^n = n(n+1)r^n$ where n is an integer. (3 marks)

5. If $\boldsymbol{\nabla}\cdot\vec{E} = 0$, $\boldsymbol{\nabla}\cdot\vec{H} = 0$, $\boldsymbol{\nabla}\times\vec{E} = -\frac{\partial H}{\partial t}$, $\boldsymbol{\nabla}\times\vec{H} = \frac{\partial E}{\partial t}$, then show that $\vec{E}$ and $\vec{H}$ satisfies wave equation of the form $\boldsymbol{\nabla}^2 u = \frac{\partial^2 u}{\partial t^2}$. (5 marks)

6. (i) Define a Hermitian matrix. (2 marks)

(i) Prove that all the eigen values of a hermitian matrix are real. (3 marks)

7. (i) Verify whether the following matrix is diagonalizable:

$$X = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

(ii) Verify whether the vectors $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -8 \\ 1 \end{pmatrix}$ forms a basis set for 3-dimensional real space. (3 marks)

8. Prove that if A and B are Hermitian matrices,

(i) $AB + BA$ is Hermitian

(ii) $AB - BA$ is Skew Hermitian. (5 marks)

9. Prove $\boldsymbol{\nabla} \times \left(\boldsymbol{\nabla} \times \vec{A}\right) = -\boldsymbol{\nabla}^2 \vec{A} + \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{A}\right)$ (5 marks)

10. (i) What do you mean by an irrotational vector. (1 marks)

(ii) Find out the values of a, b and c for which the $\vec{A} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$ will be irrotational. (4 marks)

11. Consider the following: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\ln x$

(i) Explain with reason if the given equation is linear and homogeneous. (2 marks)

(ii) Find out the general solution of it. (3 marks)

12. Consider the following equation: $e^{-x^2}\frac{dy}{dx} - \left(2xye^{-x^2} + xe^{-x^2}\right) = 0$

(i) Explain with reason whether the equation is exact. (2 marks)

(ii) Find out the general solution of the equation. (3 marks)